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On the exact mass spectrum of quarks

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Dedicated to Prof. I. Prigogine on the occasion of his 85th Birthday

Abstract

The present work proposes a general methodology for obtaining the exact $\mathscr{E}^{(\infty)}$ mass spectrum of current and constituent quarks. The theory is based upon a physical interpretation of four-dimensional fusion algebra, noncommutative geometry and related theories. All the so-obtained results were found to be in remarkable agreement with the generally accepted experimental and theoretical results found in the literature. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

As is well known and despite great effort, no one has ever been able to observe beyond any doubt a free quark [1,2]. As a consequence of this fact which is termed quarks confinement, one cannot measure their masses directly and all the data available in the literature are based partially or indirectly on qualitative measurements and theoretical arguments [2]. As a result of this situation there are two seemingly conflicting types of masses for the quarks which one can stipulate, the constituent mass and the current mass.

In the present work we present what we believe to be a novel view of the problematic of the mass spectrum. Put in the most simplistic terms we can say that both the current and the constituent masses are needed for a proper understanding of the mass spectrum of quark. Consequently we conjecture that the mass spectrum is in fact a reflection of the quantum topology of space time which essentially defines it via the formalism of four-dimensional fusion algebra and other alternative theories such as Connes noncummatative geometry, $\mathscr{E}^{(\infty)}$ theory and Freedman Surface of the gropes of four-dimensional topological manifolds [3]. This conclusion may be reinforced by the oscillation of a surprisingly simple classical mechanical system with golden mean frequencies as shown in the conclusion at the end of this paper.

2. Fusion algebra and the A_4 fusion

A fusion algebra is a finite-dimensional associative, unital, involutive algebra A over C equipped with a distinguished basis $B = B_A$, such that the structure constants

$$\{N_{XY}^Z : X, Y, Z \in B\} \text{ defined by } XY = \sum_{Z \in B} N_{XY}^Z Z$$

satisfies the following conditions

(1) $N_{XY}^Z \in Z_+ \; (= \{0, 1, 2, \ldots\}) \; \forall X, Y, Z,$

(2) the identity, denoted simply by 1 of the algebra A belongs to B, then

$$N_{X1}^Y = N_{1X}^Y = \delta_X^Y \quad \forall X, Y \in B,$$

(3) B_A is closed under the involution of A, hence there exists an involution

$$B \ni X \mapsto \tilde{X} \in B \quad \text{such that } \tilde{X} = X^* \quad \forall X \in B,$$

(4) $N_{XY}^Z = N_{\tilde{X}Z}^Y \quad \forall X, Y, Z \in B.$

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Table 1
Mass of quarks

Constituent quarks, m^* (GeV)	Conjectured experimental value (GeV)	Current quarks, m (GeV)	Conjectured experimental value (GeV)
$m_u^* = 0.3365247587$	~0.336 (up quark)	$m_u = 0.005236067977$	~0.005–0.0056
$m_d^* = 0.33888538$	~0.338 (down quark)	$m_d = 0.008472123594$	~0.008–0.01
$m_s^* = 0.469787138$	~0.5 (strange quark)	$m_s = 0.179442719$	~0.175
$m_c^* = 1.520263114$	~1.5 (charm quark)	$m_c = 1.270820393$	~1.27
$m_b^* = 4.69787138$	~4.6 (bottom quark)	$m_b = 4.236067977$	~4.25
$m_t^* = 179.442711$	~180 (top quark)	$m_t = 42.36067977$	~40–45

Note that the ratios of constituent and current masses of the quarks define the dimension function of the four-dimensional fusion algebra. Thus we have for $m_u^* \simeq m_d^* = 0.33888538$ the following exact relations which define $\mathscr{E}^{(\infty)}$ space as well as the noncommutative geometry of Connes and Von Neumann's dimensional function of his continuous points less geometry and Freedman Surface of the gropes of four-dimensional manifolds: $m_d^*/m_u^* = 1$, $m_d/m_u = 1/\phi = 1+\phi$, $m_c^*/m_s^* = 3+\phi^3$, $m_b^*/m_t^* = (\phi^{-2})(100^{-1})$, $(m_c/m_s)^* + (m_d/m_u)^* = 4+\phi^3$, $(m_d/m_u) + (m_u/m_d) = 2+\phi^3 = \sqrt{5}$ and $\{2[(m_c/m_s)^* + (m_d/m_u)^* + (m_d/m_u)]\}^2 = [2(5.85410101966)]^2 = 137 + k_0 = 137.082039325 \equiv \bar{\alpha}_0$. Note also that the so called experimental values in the case of constituent quarks are those inferred from models of hadrons spectrum.

Table 2
Quarks current mass (m) in GeV

Colour	Mass in terms of $\phi = (\sqrt{5}-1)/2$ (GeV)	Mass in terms of the coupling constants $\bar{\alpha}_i$ (MeV)	Mass in terms of other fundamental particles' mass
u (up)	$(1/\phi)^3 + 1 = (1/\phi)^3(1+\phi^3)$ $= 5.236067$ $= 0.005236067$	$(\bar{\alpha}_{gs}/5) = 5 + \phi^3$ Note: $m_u + m_d = (\bar{\alpha}_0)/10$ $= 13.7082$ MeV	$(100)(m_e)^2/5 = (20)m_e^2$ Note: $m_u + m_d = \dfrac{2m_n}{\langle m_\Pi \rangle}$
d (down)	$(2)(1/\phi)^3 = (4.23606)(2)$ $= 8.47212$ $= 0.00847212$ Note: $m_u + m_d = (2)(1/\phi)^4$ MeV	$(\bar{\alpha}_g/5) = 8.4721$ MeV, where $\bar{\alpha}_g = 42 + 2k$ and $k = \phi^3(1-\phi^3)$ $= 0.18033989$	$[(1+\phi)(100)(m_e)^2]/5 = \left(\dfrac{1}{\phi}\right)(20)m_e^2$ where $m_e = (\sqrt{\bar{\alpha}_{gs}})/10$ $= \sqrt{26+k}/10$ $= 0.51166$ MeV
s (strange)	$(10)(1/\phi)^6 = 0.1794427193$	$\dfrac{1}{2}(\bar{\alpha}_{gs})\left(\dfrac{\bar{\alpha}_0}{10}\right)(\bar{\alpha}_{gs})(\bar{\alpha}_0)/20$ MeV $= (\bar{\alpha}_g)^2/10$	$\dfrac{(m_e)^2}{2}(\langle m_\Pi \rangle)(10)$
c (charm)	$(30)[(10)(1/\phi)]^3 = 3[(10)(1/\phi)]^3/10$ $= 1.2708$	$(\bar{\alpha}_{ew})(10) = 30(\bar{\alpha}_g) = \dfrac{(\bar{\alpha}_0 - 10)}{100}$	$(\langle m_\Pi \rangle - 10)(10) = 3m_b/10$ $= 1.270820393$ GeV
b (bottom)	$(10)(1/\phi)^3 = 4.236067977$ GeV	$(10)^2(\bar{\alpha}_g)$	$\left(\dfrac{m_c}{3}\right)(100) = \left(\dfrac{\langle m_\Pi \rangle}{3+\phi^3}\right)\Big/10$ GeV
t (top)	$(10)^4(1/\phi)^3 = 42.36067977$, where $\phi = (\sqrt{5}-1)/2$ $= 0.618033989$	$(10)^3(\bar{\alpha}_g)$	$\left(\dfrac{m_c}{3}\right)(10)^3$ Note: $(42.3606)^2/10 = 179.44274$ $(m_t)^2/10$ MeV $= 179.44 = m_t^*$ GeV

Note: m_e is the mass of the electron (0.511 MeV), $\bar{\alpha}_0$ is the electromagnetic fine structure constant (137.082039325), $\bar{\alpha}_{ew}$ is the fine structure constant at the electroweak scale (127.0820393), $\bar{\alpha}_g$ is the GUT Coupling Constant (42.36067977) and $\bar{\alpha}_{gs} = 26.18033989$, $\langle m_\Pi \rangle$ is the average mass of the Π meson (pion) in MeV ($\langle m_\Pi \rangle = 137.082039325$).

Table 3
Quarks constituent mass (m^*) in GeV

Colour	Mass in terms of $\phi = (\sqrt{5}-1)/2$	Mass in terms of coupling constants $\bar{\alpha}_i$	Mass in terms of other fundamental particles' mass
u*	$10[(1/\phi)^7 + (1/\phi)^3 + \phi^2]$ $= (10)(1/\phi)^3\left[\frac{7+k_0}{10} + 7 + \phi^3\right]$ $= 0.3365247587$ GeV, where $\phi = 0.618033989$	$(7.94427191)\bar{\alpha}_g = 0.3365247$ Note : $(42)(8) = 336 = (42)^2/5.25 = 336$	$(3.97123)(\phi)(\langle m_\Pi\rangle) = (2.45094)(\langle m_\Pi\rangle)$, where $\langle m_\Pi\rangle = (137 + k_0)$ MeV and $k_0 = \phi^5(1-\phi^5)$
d*	$10[(1/\phi)^7 + (1/\phi)^3 + \phi]$ $= (8)(10)(1/\phi)^3$ $= 0.3388854$	$(8)(\bar{\alpha}_g)$ Note : $(26)(13) = 338 = (26)^2/2 = 338$	$8\left(\frac{\bar{\alpha}_0}{2}\right)\phi = 4(\langle m_\Pi\rangle)\phi$, where $\bar{\alpha}_0 = 137.0820393 = \langle m_\Pi\rangle$
s*	$(6+k)(1/\phi)^9 = (10)(1/\phi)^8$ $= 0.46978$ GeV	$(\bar{\alpha}_0)^2/40 = \frac{m_n}{2} = \frac{939.574}{2}$ $= 0.46978$ GeV	$(m_n/2) = (m_s)(m_e)^2(10)$, where m_n is the mass of the neutron $m_n = (\bar{\alpha}_0)^2/20 = 939.574$ MeV
c*	$(2)(6+k)(1/\phi)^{10} = 2(10)(1/\phi)^9$ $= 1.5202$ GeV	$2(\bar{\alpha}_g)^3\left(\frac{1}{100}\right) = (\bar{\alpha}_g)^3/50$	$(m_n)(1/\phi) = 2(m_s)(m_e^2)(10)$
b*	$(4-k)(6+k)(1/\phi)^{11} = (10^2)(1/\phi)^8$ $= 4.6978$	$(\bar{\alpha}_0^2)/4 = 4697.871$ MeV $= 4.697871$ GeV	$(m_n/2)(10) = m_n/5 = (m_s)(m_e^2)(10)^2$
t*	$(4-k)^2(6+k)^2(1/\phi)^{12}$ $= (10)(1/\phi)^6$ $= 179.442711$ GeV, where $k = \phi^3(1-\phi^3)$ and $\phi = (\sqrt{5}-1)/2$	$(10)^2\left(\frac{1}{2}\right)(\bar{\alpha}_0)(\bar{\alpha}_{gs})$ $= [(\bar{\alpha}_g)^2/10]/1000$ MeV $= (\bar{\alpha}_g)^2(10)^2$ GeV	$(m_s)(10)^3 m_t^* = (m_t)^2/10$ $= (42+2k)/10$ $= 179.442711$ GeV

Table 4
The ratios of quark's masses

Constituent quarks	Current quarks
$\left(\frac{m_u}{m_d}\right)^* = 1$	$\left(\frac{m_u}{m_d}\right) = \phi$
$\left(\frac{m_s}{m_c}\right)^* = 1/(3+\phi^3) = 0.30901699$	$(m_s/m_c) = \frac{1}{7+k_0}$; $k_0 = \phi^5(1-\phi)^5$
$\left(\frac{m_b}{m_t}\right)^* = (1+\phi)^2/100$	$(m_b/m_t) = 0.1$
$\left(\frac{m_d}{m_u}\right)^* = 1$	$(m_d/m_u) = \phi + 1$
$\left(\frac{m_c}{m_s}\right)^* = 3 + \phi^3$	$(m_s/m_c) = 7 + k_0$; $k_0 = 0.0820393$
$\left(\frac{m_t}{m_b}\right)^* = \phi^2(100) = 38.1966011$	$(m_b/m_t) = 10$

Table 5
Intrepretation of different quarks mass ratios in terms of four-dimensional fusion algebra, noncommutative geometry and related theories

Quarks mass ratio	Fusion algebra and related theories
$\left(\frac{m_u}{m_d}\right)^* = \left(\frac{m_d}{m_u}\right) = 1$	$d(1) = d(\varepsilon) = 1$ or $d_c^{(1)} = \mathrm{Dim}\,\mathscr{E}^{(1)} = 1$
$\frac{m_u}{m_d} = \phi$	$1/d(\alpha) = 1/d(\beta) = \phi$ or $d_c^{(0)} = \mathrm{Dim}\,\mathscr{E}^{(0)} = \phi$
$\left(\frac{m_d}{m_u}\right) = 1/\phi = 1 + \phi$	$d(\alpha) = d(\beta) = 1/\phi$ or $K_0(A)^+ = \left(\frac{1+\sqrt{5}}{2}\right)a + b = 1/\phi \quad \text{for } a = b = 1$ or $d_c^{(2)} = \underset{H}{\mathrm{Dim}}\,\mathscr{E}^{(2)} = 1/\phi$
$\left(\frac{m_c}{m_s}\right)^* = 3 + \phi^3$	$D'(a) = \gamma_1(D(a)) + \gamma_2 = 3 + \phi_3 \quad \text{for } D(a) = \phi \quad \text{and} \quad \gamma_1 = \gamma_2 = l$
$\left(\frac{m_c}{m_s}\right)^* + \left(\frac{m_d}{m_u}\right)^* = 4 + \phi^3$	$(1/\phi)^3 = \langle d_c \rangle = 4 + \phi^3 = \bar{\alpha}_g/10$, where $\langle d_c \rangle$ is the expectation value of the Hausdorff dimension of $\mathscr{E}^{(\infty)}$ and $\bar{\alpha}_g = 42.36067977$ is the Grand unification–quantum gravity coupling constant in the nonsupersymmetric case.
$\left[2\left[\left(\frac{m_c}{m_s}\right)^* + \left(\frac{m_d}{m_u}\right)^* + \left(\frac{m_d}{m_u}\right)\right]\right]^2$ $= [(2)(5.854101)]^2 = 137.0823$	The inverse electromagnetic coupling constant $\bar{\alpha}_0 = 1/\alpha_0 = 137 + k_0 = 137.082039325$

The algebra which will be used here is a specific fusion algebra, namely the four-dimensional fusion algebra A_4. This is a four-dimensional fusion with $B_A = \{1, \alpha, \beta, \varepsilon\}$ and the matrices corresponding to left multiplication by the basis vectors are [3,4]

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad L_\beta = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$L_\alpha = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad L_\varepsilon = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The symmetry of the matrices shows that the involution is trivial when restricted to the basis B. The most important result for the present work then follows as the dimension function given by [3,4]

$$d(1) = 1,$$

$$d(\varepsilon) = 1,$$

$$d(\alpha) = 1/\phi = 1 + \phi,$$

$$d(\beta) = 1/\phi = 1 + \phi,$$

where $\phi = (\sqrt{5} - 1)/2$ exactly as in $\mathscr{E}^{(\infty)}$ and Penrose's tiling space according to Conne's theory of noncommutative geometry [3].

3. The quarks mass spectrum

To avoid confusion it is advisable to use a slightly different notation for the current quarks and constituent quarks which we distinguish by asterisks. Now we make the first striking and most important identification relating four-dimensional fusion algebra with the mass spectrum of the quarks, namely that (see Tables 1–5)

$$\left(\frac{m_d}{m_u}\right)^* = 1$$

and

$$\left(\frac{m_d}{m_u}\right) = 1/\phi = 1 + \phi.$$

Table 6
The mass of current quarks in terms of m_n

Colour	Mass	Remarks
s	$m_s = (m_n)(\phi^2/2) = 179.427193$ MeV (usually conjectured value 175 MeV)	$\bar{\alpha}_0\phi^2 = 52 + 2k = 2\bar{\alpha}_{gs}$ $m_s = (m_n)\dfrac{10}{2\bar{\alpha}_{gs}}$
c	$m_c = (\sqrt{m_n})(\bar{\alpha}_g - 5k) = (\sqrt{m_n})(41.45898034)$ $= (m_n)(1.352549156) = (1270.820393)$ MeV $= (\bar{\alpha}_0 - 10)(10)$ MeV	$m_c = (\bar{\alpha}_0 - 10)(10)$ MeV Thus $\bar{\alpha}_0 = \left(\dfrac{m_c}{10} + 10\right)$
b	$m_b = (\sqrt{m_n})(1 + \phi^2)(10) = (\sqrt{m_n})(\bar{\alpha}_0 + 1.1145618)$ $= (\sqrt{m_n})(138.1966012) = 4236.67977$ MeV $= (m_n)(4.508497189) = (m_n)(4 + \phi^3 + 0.272429)$	$m_b = (\bar{\alpha}_g)(100)$ $\dfrac{m_b}{100} = \bar{\alpha}_g$ MeV (usually conjectured value: $m_b = 4.25$ GeV)
t	$m_t = (\sqrt{m_n})(1 + \phi^2)(10) = m_n(\bar{\alpha}_g + 2.72429)$ $= 42.3606$ GeV	$m_t = (\bar{\alpha}_g)(1000)$ $\bar{\alpha}_g = \left(\dfrac{m_t}{1000}\right)$ MeV (usually conjectured value 40–45 GeV)
(u,d)	$m_u + m_d = (m_n)(2/137.0820393) = \bar{\alpha}_0/10$ $= (m_n)(0.1458980) = (2m_n)/(\bar{\alpha}_0)$ (usually conjectured value, $m_u = 5$–5.6 MeV, $m_d = 8$–10 MeV)	$(m_n)/179.4426 = m_u$ $\quad (m_n)/m_s = m_u$ $= 5 + \phi^3$ MeV $(m_n)/110.90166 = m_d$ $\quad m_n/\bar{\alpha}_{ew} = m_d$ $= 8.4721$ MeV

Note: m_n is the mass of the neutron (939.5742749 MeV).

In other words

$$d(1) = d(\varepsilon) = \left(\frac{m_d}{m_u}\right)^* = 1$$

and

$$d(\alpha) = d(\beta) = \frac{m_d}{m_u} = 1/\phi = 1 + \phi.$$

Using the above identification it is a straight forward matter to calculate the entire "known" mass spectrum of quarks and the result, as can be seen from Table 1, is in excellent agreement with that found scattered in the literature using different ad hoc methods and indirect qualitative measurements [2]. In particular we display in Tables 2–5 our results written in three different forms, namely in terms of the four-dimensional fusion algebra function as well as in terms of the different coupling constants and the mass of other fundamental particles. Also noteworthy is the relation between the different mass ratios and the expectation value of the Hausdorff dimension of $\mathscr{E}^{(\infty)}$ as well as the fine structure constant α_0 which are summarised in Table 5.

4. The mass spectrum of the elementary particles of high energy physics

It is a truly remarkable fact that not only can we determine the mass spectrum of quarks using the proposed theory but we can also reproduce accurately the masses of all known elementary particles [2] as can be seen from Table 8. These results are closely related to the fact that the mass spectrum of quarks is easily expressed in terms of

Table 7
The mass of constituent quarks in terms of m_n

Colour	Mass m^*	Remarks
s*	$m^* = (m_n)(0.5) = 469.78713$ MeV, where $m_n = (\bar{\alpha}_0)^2/20 = 939.574$ MeV	$m_s^* = \frac{1}{2}(\alpha_0^2/20) = \bar{\alpha}_0^2/40$ Experimental value: $m_s^* = 0.5$ GeV
c*	$m_c^* = (m_n)(1+\phi) = 1520.263112$ MeV	$m_c^* = \frac{(\bar{\alpha}_0)^2}{20}(1+\phi) = (\bar{\alpha}_0)^2/(2(6+k))$ Experimental value (indirect) $m_s^* \simeq 1.5$ GeV
b*	$m_b^* = (m_n)(5) = 4697.8713$ MeV	$m_b^* = [(\bar{\alpha}_0)^2/20](5) = (\bar{\alpha}_0)^2/4 = (10)(m_s^*)$
t*	$m_t^* = (m_n)(5)(10)(4-k) = 179442.7187$ MeV $= 179.442$ GeV	This formula involves 2 of the Heterotic strings dimensions: $(4-k) \simeq 4$ and 10 while (5) is a $\mathscr{E}^{(\infty)}$ dimension $5 = 4+1$. Experimental value 180 GeV (D. Perkins and Steven Weinberg)
d*	$m_d^* = [\sqrt{m_n}\sqrt{5} - 1]/200 + \frac{1+k}{1000} = 0.33888538$ GeV $= 338.88538$ MeV $= (1/\phi)^3(80) = 338.88538$ MeV	We could write this as following $\begin{bmatrix} m_d^* \\ m_u \end{bmatrix} = \begin{bmatrix} [\sqrt{m_n}\sqrt{5}-1]/200 + \frac{1+k}{200} \\ [\sqrt{m_n}\sqrt{5}-1]/200 - \frac{1+k}{200} \end{bmatrix}$
u*	$m_u^* = [\sqrt{m_n}\sqrt{5} - 1]/200 - \frac{1+k}{1000} = 0.3365247587$ GeV $= 336.5247587$ MeV	Experimental value (indirect) $m_u^* = 0.336$ $m_d^* = 0.338$ Within the general $\mathscr{E}^{(\infty)}$ theory we must have $m_d^* = m_u^* = (1/\phi)^3(80) = 338.88538 = (8)(\bar{\alpha}_g)$

Table 8
Mass of subatomic particles, resonance and gauge bosons

Particle	Theoretical mass	Experimental value
e (Electron)	$\sqrt{\bar{\alpha}_{gs}}/10 = 0.51166$ MeV; $m_e(\min) = \sqrt{26}/10 = 0.5099$	0.511 MeV
n (neutron)	$(\bar{\alpha}_0)^2/20 = 939.574249$ MeV	939.563 MeV
P (Proton)	$(\bar{\alpha}_0 - k_0)/20 = (137)/20 = 938.45$ MeV	938.27231 MeV
$\Pi^{\pm}$(Π meson)	$\bar{\alpha}_0 + (5/2) = 139.5820393$ MeV	139.57 MeV
Π^0	$\bar{\alpha}_0 - (5/2) = 134.5820393$ MeV	134.98 MeV
$\langle\Pi\rangle$	$\frac{1}{2}(m_{\Pi^{\pm}} + m_{\Pi^0}) = \langle m_{\Pi}\rangle = 137 + k = 137.0820$ MeV	137.275 MeV
$K^{\pm}$ (Kaon)	$(\text{Dim } E_8 \otimes E_8) - 2 = (496 - k^2) - 2 = 493.967$ MeV	493.646 MeV
K^0	$(496 - k^2) + 2 = 497.967$ MeV	497.671 MeV
$\langle K\rangle$	$\frac{1}{2}(K^{\pm} + K^0) = \langle m_K\rangle = 496 - k^2$ MeV $= 495.967$	495.67 MeV
$\Delta(1232)$	$(m_{\Pi^0} + 4\phi)(9) \simeq (137)(9) \simeq 1233$ MeV	1230–1234 MeV
Ω^-	$(10)[\bar{\alpha}_0 + (49)(\phi)] = 1673.657047$ MeV	1672.43
Exi$^-$	$10[\bar{\alpha}_0 - (8)(\phi)] = 1321.377$ MeV	1321.32
Exi^0	$10[\bar{\alpha}_0 - (9)(\phi)] = 1315.19733$ MeV	1314.9
τ (tau)	$[10(m_s)] - [(m_s)/10] = 1776.4829 \simeq (42)^2 + (26/2) = 1777$ $(10)(179.4427) - \frac{179.4422}{10} = m_{\tau}$	1784.1 (Donald Perkins gives 1777)
η	$[(4)(\alpha_{gs})]^2/20 = 548.3281574$ MeV	548.8 MeV
Σ^+	$\frac{1}{1000}\left[\frac{(\bar{\alpha}_{ew})(m_n)}{100}\right] - \left(\frac{1+\phi}{10}\right)^3 = 1.18979$ GeV	1.18937 GeV
Σ^0 (sigma)	$\frac{1}{1000}\left[\frac{(\bar{\alpha}_{ew})(m_n)}{100}\right] + (2/3)\left[\frac{1+\phi}{10}\right]^3 = 1.196854$ GeV	1.19743 GeV
Σ^-	$\frac{1}{1000}\left[\frac{(\bar{\alpha}_{ew})(m_n)}{100}\right] - (1/3)\left(\frac{1+\phi}{10}\right)^3 = 1.192618$ GeV	1.19255 GeV
$\langle\Sigma\rangle$	$\frac{1}{1000}(\bar{\alpha}_{ew}/100)(m_n) = 1.194030149$ GeV	1.19328 GeV
μ (meuon)	$m_{\mu} = \sqrt{(1/\phi)^5(10)^3} = 105.3098759 \simeq (20 + k)(5 + \phi^3) = 105.665315$	105.65839 MeV
m_{μ}/m_e	$\sim (10)\frac{105.5728}{\sqrt{26 + k}} = 206.33097$ $\simeq \left(\frac{\text{Dim } E_8 \otimes E_8}{\text{Dim SU}(5)}\right)(10) = \left(\frac{496}{24}\right)(10) = 206.666$	Experimental value is 206.768262
W	$(\bar{\alpha}_0)(1 - \sin^2\theta_w)^2(10)^3 = 80$ GeV, $\sin^2\theta_w = \phi^3$	80.4 GeV
Z	$\frac{m_w}{\sqrt{1 - \phi^3}} = 91.5298244$ GeV	91.188 GeV
η'	$(m_{\eta})(7/4) = [(4)(\bar{\alpha}_{gs})]^2\frac{7}{80} = 959.5742755$ MeV	957.5 MeV
γ_{1s}	$(4236.067977)\sqrt{5} - 2(6 + k) = 9459.775272 = (m_b)(\sqrt{5}) - 2(6 + k)$ $= (m_b)\sqrt{5} - 20\left(\frac{m_d}{m_u}\right) = 9459.775272$ MeV	9460.3 MeV

Table 8 (Continued)

Particle	Theoretical mass	Experimental value
Λ	$(m_{\Pi^\pm})(8) = (139.58)(8) = 1116.6$ MeV	1115.63 MeV
$J/\psi_{(1s)}$	$(m_{\Pi^\pm})(22+k) = 3095.931842$ MeV	3096.9 MeV
$\rho(770)$	$(5+\phi)(\langle m_\Pi \rangle) = 770.1315561$	770 MeV
$\omega(783)$	$(m_{\Pi^\pm})(5+\phi) = 784.131$ MeV	782.0 MeV
η_0	$(496-k^2)(6) = 2975.804$	2979.6 MeV

the mass of the neutron as shown in Tables 6 and 7 and this in turn is related to the unification of all fundamental forces as will be explained elsewhere [1] (see Table 8).

5. Conclusion

Based on four-dimensional fusion algebra and related theories, we produce the entire mass spectrum of the quarks and indicated that the same method may be used to generate the mass of all known elementary particles.

At least in the case of quarks it is conjectured that what we consider to be a "real" particle is in "reality" a highly localised vibration, a standing wave simulating a particle. This may be made plausible by considering a simple two-masses coupled classical oscillator. Setting $C = m = 1$, the frequency of joint vibration is easily found to be

$$\omega_1 = \sqrt{(3+\sqrt{5})/2} = 1/\phi \quad \text{and} \quad \omega_2 = \sqrt{(3-\sqrt{5})/2} = \phi.$$

In other words

$$\omega_1 = m_d/m_u$$

and

$$\omega_2 = m_u/m_d.$$

In fact using Tables 1–5 it can be easily shown that $(m_b/m_t)^* = (1/\phi)^2/100$ and that

$$\bar{\alpha}_0 = \left[\left(\frac{m_c}{m_s}\right)^* + \left(\frac{m_s}{m_d}\right)^* + \left(\frac{m_s}{m_c}\right) + \left(\frac{m_d}{m_t}\right)^*\right]^2 = 137.036 \simeq \bar{\alpha}_0 \text{ (Experimental)}.$$

The reader is also referred to a related work by Sidharth [5] where he came to similar conclusions.

References

[1] Weinberg S. The quantum theory of fields, vol. II. Cambridge, 1996.
[2] Frauenfelder H, Henley EM. Subatomic Physics. Englewood Cliffs, NJ: Prentice-Hall; 1991.
[3] El Naschie MS. Wild topology, hyperbolic geometry and fusion algebra of high energy particle physics. Chaos, Solitons & Fractals 2002;13:1935–45.
[4] Kodiyalam V, Sander VS. Topological quantum field theories from subfactors. London: Chapman & Hall; 2001.
[5] Sidharth BG. Chaotic universe. New York: Nova Science Publishers; 2001.

[1] A most striking example for the deep connection between the topology of $\mathcal{E}^{(\infty)}$ and transfinite heterotic string theory and the masses of elementary particle is the following relation

$$(10)(\text{Dim } E_8 \otimes E_8 \mid_{\mathcal{E}^{(\infty)}})/(\text{Dim SU(5) of GUT}) = \left(\frac{m_\mu}{m_e}\right) = 206.6565035$$

where m_μ is the mass of the muon $m_\mu = (20+k)(5+\phi^3)$ and m_e is the mass of the $\mathcal{E}^{(\infty)}$ electron $m_e = (\sqrt{\bar{\alpha}_{gs}})/10 = 0.51166$ MeV. Note that Dim $E_8 \otimes E_8 = 496 - k^2$ and Dim SU(5) = 24 while the experimental value for $(m_\mu)/m_e$ is 206.768262 which is remarkably close to our theoretical prediction.