Quantum Collapse of Wave Interference Pattern in the Two-slit Experiment : A Set Theoretical Resolution

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Abstract

The paper gives a strict mathematical justification and physical explanation of the disappearance of interference patterns when which way information is obtained in the two-slit experiment with quantum particles. Furthermore we give a realistic and causal explanation of the quantum state vector reduction on measurement and the associated entropy decrease without invoking any black hole physics.

Keywords: E-infinity, noncommutative geometry, number theory in physics, quantum sets, golden mean in quantum mechanics, Ji-Huan He's eleven dimensional fractal spacetime, quantum measurement, E-infinity entanglement probability.

1. Introduction

The present work presents what we consider to be a rather convincing resolution of the which way information paradox of the two-slit experiment [1-4] based on random fractal sets and the extended Menger-Urysohn transfinite theory of dimension [6-11]. We use for each set two component dimensions [12-14]. The first component is an invariant topological dimension and the second component is a corresponding Hausdorff-fractal dimension calculated using a continuous dimensional function [15]. Starting from A. Connes function of Penrose noncommutative tiling [9,11,15]

$$D = a + b\ \varphi\ ;\ a, b\ ;\ \varphi = (\sqrt{5} - 1)/2 \quad ,$$

we demonstrate the equivalence of this function to that of E-infinity bijection formula [9,11,14]

$$d_c^{(n)} = (1/d_c^{(0)})^{n-1}$$

where n is a real integer representing the Menger-Urysohn topological dimension and $d_c^{(0)} = \varphi = (\sqrt{5} - 1)/2$ is the topological invariant of the related dimension group [11, 14].

The golden ratio $\varphi = (\sqrt{5} - 1)/2 = 0.618033989$ which was experimentally observed rather recently in the Helmholtz Centre- Germany, in quantum mechanics [16] arises here naturally from the requirement of a random fractal horizon of a noncommutative geometry [15, 17-20] related to the compactified Klein modular curve [17,18]. This curve with 336 + 3 = 339 hierarchal degrees of freedom or isometries is also equivalent to the holographic boundary of E-infinity spacetime [11, 17]. Since Penrose tiling is essentially a quotient space, the holographic boundary as well as E8 exceptional Lie symmetry group may be regarded as a deformation of this tiling and the compactified SL (2,7) which is basic to E-infinity theory [11, 17, 18].

One of the most important steps taken in the present work is to follow a proposal [12] for the identification of the empty set with wave-like quantum probability and the true vacuum while quantum-like particles are described as a zero set. Consequently the two component dimension relevant to the particle is [12-14, 21]:

$$\dim(\text{particle set}) = P(d_{MU}, d_H) = P(0, \varphi)$$

where d_{MU} is the Menger-Urysohn dimension and d_H is the corresponding Cantorian or Hausdorff dimension. For the quantum wave on the other hand we have

$$\dim(\text{wave set}) = W(d_{MU}, d_H) = W(-1, \varphi^2).$$

Since we have identified the quantum wave which is devoid of energy matter and momentum with the empty set, which contains no elements what so ever not even zeros, but is never the less structured and possesses a Hausdorff fractal dimension, it follows then as an almost trivial conclusion that any attempt to observe the two-slit experiment while in progress will render the empty set non-empty and instantly lead to the disappearance of the pattern caused by wave interference, leaving the fractal zero sets, i.e. particles form a patternless randomly distributed dots on the detection screen as the only observable. Using the preceding conclusion it is then a relatively straight forward mathematics which leads us to the form of semi manifold which supports the preceding process. It turns out that this semi manifold is a fractal quotient manifold of the Gaussian type [32] akin to the space of Penrose fractal tiling with a combined Hausdorff dimension given by [22]

$$D_H = \frac{(\varphi)+(\varphi^2)}{(\varphi)(\varphi^2)} = 4+\varphi^3 = 4+\cfrac{1}{4+\cfrac{1}{4+\ldots}} = 4.236067977,$$

and a Menger-Urysohn dimension

$$D_M = \frac{0+(-1)}{(0)(-1)} = \infty$$

as well as an average topological dimension equal to

$$<D> = \frac{(1/2)+(1/2)}{(1/2)(1/2)} = 4.$$

In other words this manifold is nothing else but the core of E-infinity Cantorian spacetime which may be envisaged as an infinite hierarchy of concentric four dimensional cubes resembling self similar Russian dolls which is the basic picture of E-infinity spacetime [11]. It is extremely interesting at this stage to note that although E-infinity spacetime is a discretum, i.e. totally disconnected point set semi manifold, its cardinality is strictly larger than that of the continuum [15].

2. Suggested mathematical prerequisite as an introduction to the science of the infinite

There are several essential mathematical concepts and theorems which are needed for a proper understanding of the present work. It is helpful for readers not very familiar with the mathematics of the infinite to know and appreciate the following subjects:

First one needs to understand the role of Bijection which is essential for a rigorous counting and constitutes the beginning of set theory and the work of G. Cantor [36].

Second we need to understand and appreciate the method of complete induction. It is due to Pascal but if one goes back in history one will find that the Chinese may have been there first.

Next one has to look at the work and philosophy of Henri Poincaré to deepen our understanding of the complete induction method.

The fourth step is to look at the science of converging and diverging series.

Fifth, it is recommended to consider the various famous logical paradoxes. For instance Russell's self referential statement and this will eventually lead to the appreciation that fractals are self referential.

The next step may be to study the axiom of choice and look at Banach-Tarski theorem which was considered in cosmology [22]. Step seven is a careful study of Cantor's diagonal argument to show that there are at least two types of infinity, namely (1) countable and (2) non-countable infinity. Note that the second is larger than the first and that E-infinity has a cardinality larger than the continuum. Step eight may be to consider the work of Alan Turing who invented the Turing computer which is the quintessence of a computer without hardware [3].

Next in step nine one should look at number theory and particularly the concept of Dedekindian Cut and related mathematical issues. As step number ten one needs to read some of K. Gödel (1930) and P. Cohen (1963) works on the continuum hypothesis. Step eleven is to look at the work of Emile Borel which is essential at this stage. Note that a Borel Set as discussed by Wheeler was the starting point of E-infinity mathematics [9-14].

Step twelve is to attempt to connect all that to fractals and relate the Menger Sponge to the Hilbert Hotel. Step thirteen is that to get to physics one should apply all what one has learnt from the above to the work of Max Planck and his constant as well as Heisenberg and his matrices in addition to Schrödinger and his waves. That way one could arrive at E-infinity theory [29-34] via the study of the science of the infinite. A step fourteen is sometimes recommended by starting all over again from step one.

3. **The empty set – An elementary derivation of its topological dimension (-1)**

What is the dimension of a 3D cube boundary? This is a trivial question since it is clearly an area, i.e. a surface which is 2D. That means

$$3D(\text{cube}) - 1 = 2D(\text{Surface}).$$

Next we ask a second trivial question, namely what is the dimension of the boundary of a 2D surface? It is obviously a one dimensional line

$$2D(\text{surface}) - 1 = 1D(\text{line}).$$

Finally what is the dimension of the boundary of a line? This is evidently a zero dimensional point. That means

$$1D(\text{line}) - 1 = 0D(\text{point}).$$

It seems natural that by induction one could write a general expression for the above in the form [7, 8, 10, 14]

$$D(\text{boundary}) = n - 1$$

where n is the dimension of the geometrical object for which we would like to know the dimension of its boundary. This is a trivial case of induction. However what if we want to extend this formula below a point just as we usually extended it above a 3D cube? We routinely deal in higher geometry with 4D and nD cubes as discussed by Coxeter and studied thoroughly in the context of E-infinity by Ji-Huan He [30, 31]. In this case we use induction to say that the boundary of a point has a dimension [6-8, 10, 14]

$$D = D(0) - 1 = 0 - 1 = -1$$

This is the dimension of the classical empty set as deduced for the first time by P. Urysohn and studied by K. Menger [6, 14]. This step is by no means a trivial or obvious one and is only deceptively simple. The point however is that we know of no "ordinary" geometrical object for which we can use the method of complete induction and find for instance

$$D = D(-1) - 1 = -1 - 1 = -2$$

and so on until we find D = -∞ for which the Hausdorff dimension by bijection is equal to

$$d_c^{(-\infty)} = (1/\varphi)^{(-\infty-1)} = 0$$

The only exceptional geometry for which the above is true is 'non-ordinary' Cantorian geometry [9, 11]. In Cantor sets you can split a point in two by graphical computer zooming. It is the important discovery of the present paper that this transfinite set theoretical formulation can be used in quantum physics where the quantum particle is represented by the dimension doublet of the zero set $d_c^{(0)} = \phi$ namely

$$\dim P_Q \equiv (0\ ,\ \varphi\ )$$

while the quantum or ghost wave is represented by the dimension doublet of the empty set

$$\dim W_Q \equiv (-1,\ \varphi^{\ 2})$$

as we mentioned in the introduction. It is important to always keep in mind that the empty set is the 'surface', i.e. the neighbourhood of the zero set. Consequently the wave is in a sense the surface of the particle [12, 13].

4. The relation between Connes' noncommutative geometry dimension function and E-infinity bijection formula

We start by the dimension function of the noncommutative quotient space representing the well known Penrose tiling:

$$D\ (a,b) = a + b\ \varphi \quad , \quad \varphi = \frac{\sqrt{5}-1}{2} \qquad a, b \in \mathbb{Z}$$

Our aim is to show that under certain conditions this dimension function will yield the bijection formula of E-infinity

$$d_c^{(n)} = (1/\ \varphi\ )^{n-1}.$$

Let us set D_n (a, b) to be first $D_o \equiv D$ (0, 1) and $D_1 = D$ (1, 0). Subsequently we add a_i and b_i following the Fibonacci scheme as follows:

$$
\begin{aligned}
D_0\ (0, 1) &= 0 + \varphi = \varphi \\
D_1\ (1, 0) &= 1 + (0)\ \varphi = 1 \\
D_2\ (0 + 1\ , 1 + 0) &= 1 + \varphi = 1/\ \varphi \\
D_3\ (1 + 1\ , 0 + 1) &= 2 + \varphi = (1/\ \varphi\ )^2 \\
D_4\ (1 + 2, 1 + 1) &= 3 + 2\ \varphi = (1/\ \varphi\ )^3 \\
D_5\ (2 + 3, 1 + 2) &= 5 + 3\ \varphi = (1/\ \varphi\ )^4
\end{aligned}
$$

and so on. By induction we conclude that

$$D_n = (1/\ \varphi\ )^{n-1}$$

This is the bijection formula of E-infinity theory [9, 11, 17]

$$d_c^{(n)} = (1/\varphi)^{n-1} .$$

However we see that the bijection notation is more compact and economical and we know two dimensions at once; the n is the Menger-Urysohn dimension while $d_c^{(n)}$ is the Hausdorff dimension. Consequently it is easy to extend the formula to negative dimensions so that we would have for instance

$$d_c^{(-1)} = (1/\varphi)^{(-1-1)} = (1/\varphi)^{(-2)} = \varphi^{-2}$$

which is our empty set dimension binary

$$\dim(\text{empty set}) = (-1 , \varphi^2)$$

where -1 is topologically invariant Menger-Urysohn dimension while φ^2 is the Hausdorff dimension which is not topologically invariant but extremely useful. We see clearly that the totally empty set, by complete induction, must be [9, 11, 14]

$$d_c^{(-\infty)} = (1/\varphi)^{(-\infty-1)} = (1/\varphi)^{(-\infty)} = \varphi^{(\infty)} = 0 .$$

The zero set on the other hand is [9, 11, 14, 21]

$$d_c^{(0)} = (1/\varphi)^{0-1} = (1/\varphi)^{-1} = \varphi$$

as is well known and in full agreement with the dimensional function of noncommutative geometry [15].

We cannot stress enough that it is the most important conclusion of our theory that transfinite set theoretical formulation can be used in quantum physics where the quantum particle is represented by the dimension binary of the zero set $d_c^{(0)} = \varphi$, namely [9, 11, 21]

$$\dim P_Q = (0 , \varphi)$$

while the quantum, wave reminiscent of the ghost or guiding wave of Einstein and Bohm is represented by the dimension binary of the empty set [10, 14, 32]

$$\dim W_Q = (-1 , \varphi^2)$$

as mentioned in the introduction as well as section 3. Note that the similarity with the guiding wave proposal is only partial and the two theories are by no means identical. For instance in the Einstein-Bohm picture there is no wave collapse at all exactly as in the many world picture [5, 35].

5. Some philosophical implications

Humans have an inbuilt intrinsic bias in their psyche against nothingness. Although we fear nothing like nothingness, being associated with death, we still do not regard it as physically real and integrate it on a fundamental level into the foundation of physics. So far we have been content with the zero introduced by the Indians and mediated to Europe by Arab mathematicians who saved our arithmetic and number system from the unbearable heaviness of Roman numbers.

The Author feels that the empty set of the Menger-Urysohn transfinite dimensional theory can do for physics what the zero did for mathematics when we extend the empty set dim $d_{nu} = -1$ to the totally empty set dim $d_{MU} = -\infty$. It may be instructive at this point to connect a little to fundamental philosophical problems which were considered around the middle of the last century in great depth. In that respect we may mention the views of M. Heidegger laid down in his book "Sein und Zeit" and later on the fundamental and famous work of J.P. Sartre, Being and Nothingness. In the somewhat flamboyant language of Sartre, he described the embedding of nothingness in being by liking it to a worm inside an apple. At the core of being nothingness is lurking. The exact mathematical formulation of the foundation of physics could similarly not be complete or consistent without including nothingness in the form of the empty set.

$$d_{MU} = -1$$

and

$$d_c^{(-1)} = \varphi^2, \quad \varphi = (\sqrt{5} - 1)/2$$

and the totally empty set

$$d_{MU} = -\infty$$

and

$$d_c^{(-\infty)} = 0 \quad .$$

The distance between $d_{MU} = -1$ and $d_{MU} = -\infty$ is what is termed following a similar proposal by Mandelbrot the degree of emptiness of an empty set. It is highly interesting to note that while too much philosophical interrogation did not help science to start with and a separation between what is empirical and testable and what is idle deep philosophical discussion was recommendable, the situation started changing with relativity and more so with quantum mechanics. Never the less physics did not change in essence with regard to the notion of nothingness. The introduction of the empty set on such a fundamental level clearly shows that our initial reaction to philosophy was misguided. Philosophy is part and parcel of real deep science and that is probably why fundamental science used to be called, at least in England, natural philosophy while at the time of G. Cantor it was not unusual to use the word metaphysics for the same meaning.

6. The golden mean entanglement of orthodox quantum mechanics

One of the most important fundamental results which was recently obtained using E-infinity noncommutative quantum sets is a mathematical proof of the Cantorian-fractal entanglement nature of orthodox quantum mechanics. We will not discuss the derivation of this new fact in detail in the present paper. We reserve a complete mathematical derivation and full discussion for a forth coming paper. It is sufficient here to give a hint at the main idea and the final result. The derivation is mainly based upon a combination of Ji-Huan He's polytope [30] and his eleven dimensional fractal M-theory [38] as well as the extension of Witten's T-duality between negative quantum sets as an inversion of the positive quantum sets of E-infinity theory [9, 11]. In this way it is easily shown that for the fractal-self similar eleven dimensional M-theory [38] classical quantum mechanics and field theory give an exact joint probability of entanglement equal to $\varphi^3(1 - \varphi^3)/2 = k/2 = (0.18033989)/2$ where φ is the golden mean. This is exactly equal to 9.016994 percent which is very close to a classical value reported for instance by R. Penrose [3], namely 9.17 percent. It is very interesting that in the classical case of four spacetime dimensions, orthodox quantum mechanics gives a joint entanglement probability which is much larger while at the Planck energy scale at infinite resolution corresponding to infinite dimensionality, the joint entanglement probability is zero. That way we can rule out all local hidden variable theories in a rational non-spooky manner. Quantum mechanical entanglement may thus be explained intuitively as a natural consequence of a spacetime geometry and topology which is converging to a zero measure Cantor set with the golden mean as a Hausdorff dimension [30-35].

Discussion and conclusion

In the words of R. Penrose the two-slit experiment is the 'archetypical quantum-mechanical experiment'. The subject was examined with painstaking accuracy rather recently by Nobel laureate Sir A. Leggett [23-25]. The present work however takes new direction based on important mathematical development in applying noncommutative geometry and higher dimensional transfinite set theory to the foundations of quantum mechanics [12, 26-29, 35]. Let us use the same terminology of Penrose regarding the U, O and R processes and results of measurement [24, 25, 33-35]. We recall that the empty set is defacto two identical things at the very same time, namely the surface or the topological neighbourhood of the zero set as well as being the guiding quantum wave. Similarly the zero set is a Cantorian fractal point as well as the quantum particle guided by the 'ghost' wave. Compared to the particle, the wave has very large numbers of degrees of freedom and consequently very large entropy. This may be understood in a very elementary manner by recalling that the wave is the surface of the particle and it is evident that the smaller, say a sphere, the larger is the ratio between its surface area and its volume. When the volume tends to zero, the ratio will tend to infinity. This is well known from everyday experience with dissolving a cube of sugar or sugar grains in our tea and coffee. This is another intuitive way of looking at the disproportionally large difference in the degrees of freedom to the benefit of the wave vis-à-vis the particles aspect of the quantum. Now on taking measurement on this particle-wave packet we inevitably enter into the wave and consequently into the domain of the empty set. That way the empty set becomes non-empty and practically reduced or jumps to at best, a zero set. That is to say, only the particle with its minimalistic degrees of freedom corresponding to much smaller entropy than the wave becomes experimentally manifest. That is the R outcome of the state vector reduction or quantum jumps which disturbed E. Schrödinger considerably [3]. In this way we have a mathematical explanation which is both causal, deterministic and realist for which we need not dwell on other aspects and physical theories such as the many world interpretation [4, 5] or the black hole information problem [26, 27].

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