Quantum Golden Mean Entanglement Test as the Signature of the Fractality of Micro Spacetime

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Abstract

Is the world really more than four dimensional or for instance an eleven dimensional M-theory as proposed by Witten? The present Letter suggests that it is an eleven dimensional fractal. Based on this assumption it is shown that the probability of quantum entanglement is exactly equal to that predicted by L. Hardy's well known criteria. It is further argued that this is an indirect but strong indication for the reality of the Cantorian fractal nature of micro spacetime.

Keywords: El Naschie spacetime, golden quantum mechanics, entanglement

Our world is manifestly four dimensional but do we need more dimensions to describe it accurately? Maybe but let us say for a moment that we definitely need these extra dimensions and as if this is not provocative enough, let us be even more presumptuous and ask if spacetime may also be a fractal. We know very well that our present day experimental techniques cannot give direct answers to either of the questions. However the first good news is that both of the afore mentioned presumed properties of the world and spacetime are linked and inter-dependent. The second good news, which is too good to believe although it is true, is that both suspected properties are implied by a well known quantum test based on a Gedanken experiment due to Hardy[1-3].

In his 1992 paper, L. Hardy introduced an ingenious test for quantum entanglement following Bell's theorem for two entangled particles[1-5]. It is intriguing to note that the numerical result of L. Hardy's Gedanken experiment, namely 9.017 percent for entanglement[2] may also be viewed as relatively direct confirmation of the fractal nature of micro spacetime which was formally proposed in 1990 by El Naschie to be basically an infinite but hierarchal collection of elementary triadic random Cantor sets[6,7]. This conclusion is based upon two facts. The first is the body of results obtained in the context of high energy physics using the Cantorian-fractal theory[7] as well as the experimental confirmation of the involvement of the golden mean in quantum mechanical problems on a very deep level which was revealed in an article in Science relatively recently[8,9]. The second is that on careful analysis of Hardy's 9.017 percent result using a general theory it clearly transpires that the exact untruncated value, namely 9.0169945 per cent is the T-duality or inverse of the dimension of a fractal M-theory or equivalently a fractal super gravity theory with $D = 11 + (k/2)$, which could be approximated to the classical $D = 11$ theory[10-12]. However in $D = 11 + k/2$ the crucial point is that $1/D = k/2$ is a probability and this $k/2$ probability is exactly equal to that of Hardy's quantum entanglement probability by virtue of the fact that $P = k/2 = 0.090169945 = \phi^5$ as will be derived shortly in the main body of the present paper using the general theory of fractal Cantorian spacetime[6,7,10-17]. Two important results found in the literature will be focused upon besides the main result of Hardy. These are the results found based on a four dimensional spacetime leading to a joint probability of exactly 9% [1-5,18,19] and second the result of Penrose example leading to a joint probability of exactly 8.333%, equivalent to the inverse of the dimensionality of F-theory, namely $1/D = 1/12$[4].

Our main tool in calculating the probability of entanglement is the E-infinity manifold of fractal-Cantorian spacetime theory[6,7,20]. This is a probabilistic transfinite, i.e., random fractal infinite dimensional but hierarchal hyper surface whose average dimension is found using statistical mechanics and the theory of fractal Cantor sets[6,7]. It turns out that the average topological dimension of such a manifold is exactly 4 thus mimicking our real spacetime in that respect while having an additional advantage of possessing another average dimension, a so called Hausdorff dimension equal to exactly 4 plus the golden mean to the power of three, i.e. 4.236067977. Looking carefully at this manifold it is easily shown that it resembles a four dimensional manifold inside another four dimensional manifold and so on ad infinitum like a Russian doll [6,7]. Consequently the manifold in question is formally infinite dimensional. Therefore our Cantorian manifold is described not by one dimension but by three dimensions. In fact a fourth dimension related to the spreading out of the points of the manifold which is termed mathematically the spectral dimension is exactly equal to 4.019999[7].

Let us reconsider the Hausdorff dimension $4 + \phi^3 = 4.236067977$. We see that its T-duality inversion[7] is equal $1/(4 + \phi^3) = \phi^3$ which may be interpreted as the intersection probability of being in this manifold[7]. Now we look at the relation between two points of this manifold as being a surrogate for the probability of entanglement between these two points. We may then say that if the probability at a certain point is P_i then the probability at the second point must also be equal P_i and the entanglement probability is given by the intersection rule or the multiplication theorem of probability theory, namely $P_i\ P_i = (P_i)^2$. However because of the existence of uncountably infinitely many other points, the probability $(P_i)^2$ must be simultaneously influenced by ϕ^3 so that the final probability of entanglement is simply $P = (P_i)^2\ (\phi^3)$. It is easily reasoned that ϕ^3 itself was calculated from the inversion of the average dimension $<n> = (1 + P_i)/(1-P_i)$ when setting $P_i = \phi$ and find $<n> = 4 + \phi^3$. In other words $\phi^3 = (1-P_i)/ (1 + P_i) =(1- \phi)/ (1 + \phi)$. Inserting in the P one finds the final value of intrinsic topological entanglement

$$P = (\phi^2)(\phi^3) = \phi^5, \tag{1}$$

where ϕ^2 is the local probability and ϕ^3 is the global or counterfactual part of the probability. The term counterfactual[18] is the term used by philosophers to describe the influence which stems from things which did not happen but could have happened[4,18,19]. We extend this term to mean things which are not there but influence things which are there. Thus the complimentary set of the zero set representing the empty set has a profound influence on the zero set[14-17].

We recall that quantum particles may be modeled by the zero set while the wave is the empty set obtained from the zero set by a Legendre transformation and representing a non "existing" real vacuum or empty space influencing the motion of the particle. All this was explained in detail in previous publications and is not essential to consider at length here again[14-17].

We start our derivation by recalling that the Cantorian E-infinity manifold is made of a gamma distributed infinite sum of elementary Cantor sets[6,7]. This sum is given by

$$< n >= \sum_{0}^{\infty} n(d_c^{(0)})^n = (1+d_c^{(0)})/(1-d_c^{(0)})\,. \tag{2}$$

Here n is the topological inductive dimensions of the Menger-Urysohn system, $d_c^{(0)}$ is either the Hausdorff dimension of an elementary randomly constructed triadic Cantor set or the topological probability $P_i = d_c^{(0)}$ of finding a point in this Cantor set[6,7]. We note further that a quantum particle is given by $d_c^{(0)}$ for which the topological dimension is zero while the Hausdorff dimension is $d_c^{(0)}$. This is effectively the zero set [13-17]. On the other hand the wave of the quantum particle is given by the empty set $d_c^{(-1)}$ for which the topological dimension is -1 [13-17] while the Hausdorff dimension is given by the Legendre transformation of the zero set P_i (wave) = 1 − P_i (particles) = 1− $d_c^{(0)}$ [13-17].

On the other hand from the theory of four manifolds[7] we know that the dimension is

$$D_4 = \frac{1}{2d_c^{(0)} - 1} \tag{3}$$

Equating $<n>$ to D_4 one finds an algebraic equation from which $d_c^{(0)}$ is found to be equal to the golden mean. Thus $d_c^{(0)} = \phi = (\sqrt{5}-1)/2$, inserting back we find that $<n> = 4 + \phi^3$. Consequently the "counterfactual" probability influence of our Cantorian manifold is given by [6,7]

$$P = 1/<n> = 1/(4 + \phi^3) = \phi^3\,. \tag{4}$$

On the other hand the probability for two points "particles" to get entangled quantum mechanically is given by the multiplication theorem to be $(P_i)^2$ and in general for n particles (points) is $(P_i)^n$. Consequently it is clear that the total probability is given in general by

$$P(\text{entanglement}) = (P_i^2)\ (1/<n>) = (P_i^2)\ \left(\frac{1-P_i}{1+P_i}\right). \tag{5}$$

In the case of a fractal manifold we have $P_i = \phi$ and thus

$$P = (\phi^2)(\phi^3) = \phi^5\,. \tag{6}$$

There are two remarkable things which seem to have passed unnoticed for some time. The first is that ϕ^5 is the inverse of the eleven dimensional M-theory [21] in its fractal version [11,12].

$$D = 1/\phi^5 = 11 + \cfrac{1}{11 + \cfrac{1}{11 + ...}} = 11 + k/2 = (1/\phi)^5 = 11.09016992 \quad (7)$$

where $k = \phi^3(1-\phi^3)$ and $\phi = (\sqrt{5}-1)/2$.

The second remarkable observation is that $P = \phi^5$ is itself the accurate value of what was slightly inaccurately quoted in at least the initial publication on Hardy's entanglement probability, namely the famous value of 9.017%. This is nothing but $k/2$ = 0.09016994393 when truncated for simplicity and convenience to 0.0917 which obscured the point. We note with some satisfaction that the existence of a fractal-like M-theory was suspected for some time and considered for instance by Ji-Huan He[10] as well as El Naschie on several earlier occasions [11,12]. Given the fact that ϕ is intrinsic to random triadic Cantor sets and that these sets are the building blocks of the corresponding Cantorian fractal spacetime manifold[7], it becomes understandable that the golden mean features on such a fundamental level and makes its début at various unsuspected places in quantum mechanics and more generally in natural phenomena in the micro and macro worlds too numerous to count here[6-9].

For the sake of gaining further confidence let us apply the same previous general theory to first the case of a 4D manifold and second to the examples given by Penrose in his well known treaty [4].

In the case of a 4D manifold we must have $d_c^{(0)} = P_i = 3/5$. Thus inserting in $<n>$ one finds

$$<n> = \frac{1+(3/5)}{1-(3/5)} = 4. \quad (8)$$

Consequently the "counterfactual" part of the total entanglement probability is equal to $1/<n>$ = 0.25. On the other hand$(P_i)^2$ in this case is $(3/5)^2$. The total probability is thus

$$P = (3/5)^2 \ (0.25) = 0.09. \quad (9)$$

This is exactly the 9% probability quoted in Ref.[2-5]. Next we consider Penrose example[4] which is relevant for $D = 3$ consequently $P_i = d_c^{(0)} = < d_c^{(0)} > = 1/2$. Inserting in $<n>$ one finds

$$<n> = \frac{1+(1/2)}{1-(1/2)} = 3. \quad (10)$$

The manifold contribution to overall entanglement probability is thus $1/<n> = 1/3$. The $(P_i)^2$ part is consequently $(1/2)^2 = 1/4$. The total probability is thus

$$P = (1/2)^2 \ (1/3) = 1/12. \quad (11)$$

This is a probability of 8.333 percent exactly as given by Penrose[4].

We note here our spacetime topology interpretation that $P = 1/12$ belongs to the $D = 12$ of F-theory which was developed in an attempt to have an alternative to M-theory[21].

Witten's M-theory[21] is undoubtedly a quantum leap for understanding high energy physics and is a potential theory for everything. Maybe not so well known nor as grandiose is Hardy's test for quantum entanglement and nonlocality but it is our firm belief that it is not less fundamental than M-theory and shows the potential benefits for all branches of research in quantum physics which we can gain by looking deeply at the fundamental problems of orthodox quantum mechanics as recently considered by T. Palmer [22]. For this reason the not accidental meeting of M-theory[21] with Hardy's test [1-5] in a fractal spacetime setting[11,12] is a very rewarding and admittedly mildly surprising research experience and we hope that it will promote more research into fractal spacetime theory and exploring the supposedly mysterious role played by the golden mean in this area. We stress that the appearance of the golden mean on such a fundamental level as in Hardy's test for quantum entanglement is by no means the first of its kind. The golden mean plays a fundamental and central role in quantum chaos via the KAM theorem of stability in nonlinear dynamics [6,7]. In addition it was recently found experimentally in the Helmholtz Institute in connection with some fundamental problems of quantum mechanics and ferromagnetism reported in 2010 for the first time in Science [8,9].

It may be of some historical interest to note that the first explicit connection between quantum spacetime and the golden mean as well as triadic random Cantor sets was noticed by El Naschie in 1991. Hardy found his famous result which followed from his Ph.D. thesis in 1993. Finally in Ref. 14 in section 6, El Naschie established the connection between Hardy's work and fractal eleven dimensional M-theory.

In a ten dimensional string theory we have 6 compactified dimensions plus four spacetime dimensions as in Einstein's theory of relativity. In an eleven dimensional spacetime theory the situation there is subtly different since the four dimensions of Einstein are extended as in all Kaluza-Klein theories to five dimensions[21]. However one of these five is again compactified so that our theory is strictly speaking not 6 +5 =11 dimensions but 7+4=11 dimensions where the seven are interpreted normally as being the dimensionality required for embedding the SU(3) SU(2) U(1) of the standard model. The situation for a fractal eleven dimensional theory is again very similar but requires some delicate arguments and careful analysis. To start with the obvious dimensional equation 11 – 7 = 4 is in this case modified using a general theory[6,7] to $(11+\phi^5)-(7-\phi^4)=4+\phi^3$ where $\phi=(\sqrt{5}-1)/2$ is the golden mean, $4+\phi^3$ = 4.236067977 is the Hausdorff dimension of the Hilbert cube and ϕ^5 = 0.09016994 is Hardy's probability for quantum entanglement. It is also equal $k/2 = \phi^5$ where $k = \phi^3(1-\phi^3)$ which is one of the important values of the naturally quantized golden mean number system [6,7]. For instance the exact quantized values corresponding to the classical values of the dimension of certain important Lie symmetry groups for instance │E8 E8 │= 496 and│SL(2,7)│= 336 are 496 – (k^2/2) and 336 + 16k as explained in detail for instance in Refs.[6,7]. From the preceding analysis and discussion it seems to be a natural logical conclusion that involvement of the golden mean in quantum mechanics can be traced back to the Cantorian-fractal geometry and topology of micro spacetime. The agreement of Hardy's test probability with the eleven dimensional fractal topology of spacetime could never be a mere coincidence of any type unless Einstein was wrong in saying that the Lord is subtle but not malicious. Nature is definitely subtle but it is equally definitely not maliciously misleading to the extent of inventing all these highly unlikely coincidences only to lead us astray. Confidence in reason precludes such a thing.

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